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CENTRE DE RECHERCHE SUR LES TRANSPORTS
UNIVERSITÉ DE MONTRÉAL
C.P. 6128, SUCCURSALE A.
MONTRÉAL
CANADA H3C 3J7

TÉLÉPHONE: (514) 343-7575
TÉLEX: 05-24146 BIBPOLYTEC
EMAIL: crt@crt.umontreal.ca
TÉLÉCOPIEUR (FAX): (514) 343-7121

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CENTRALIZED AND DECENTRALIZED STOCHASTIC MODELS IN TELECOMMUNICATION NETWORKS

by

Brunilde Sanso*
François Soumis**
Michel Gendreau*

* Centre de recherche sur les transports, Université de Montréal, C.P. 6128, succursale A, Montréal (Québec) H3C 3J7.

** École des Hautes Études Commerciales, Groupe d'études et de recherche en analyse des décisions (GERAD) et École Polytechnique, 2500 chemin de Polytechnique, Campus de l'Université de Montréal, Montréal (Québec) H3C 3J7.

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ABSTRACT

We present and compare two routing models characterized by two different routing policies for circuit-switched telecommunications networks. The first model is for a centralized optimization routing policy whereas the second model obeys to a decentralized type of routing. Both models are formulated so that the stochastic character of the demand is taken into account. The two models are solved by the same type of algorithm, a convex simplex implementation, adapted differently according to the model. Comparative results between the two policies are discussed.

Key words: telecommunications networks, circuit-switching, routing models, centralized and decentralized policies.

RÉSUMÉ

Nous présentons et comparons deux modèles d'acheminement pour les réseaux de télécommunications à commutation par circuits. Ces modèles correspondent respectivement à des politiques d'acheminement centralisée et décentralisée. Le caractère stochastique de la demande est pris en compte explicitement dans les deux formulations. Ces modèles sont résolus par des versions spécialement adaptées de l'algorithme du simplexe convexe. Nous terminons par une discussion comparative des résultats obtenus sous les deux politiques d'acheminement.

Mots-clés: réseaux de télécommunications, commutation par circuits, modèles d'acheminement, politiques centralisée et décentralisée.

1 Introduction

Telecommunications is one of the technological areas which have undergone dramatic changes in recent years. In fact, the introduction of new technologies in this field has changed the whole philosophy of telecommunication network design, call routing and network management. In particular, switching techniques have become much more flexible. Since telecommunication routing policies have always been determined by the restrictions established by the available switching technology, this new flexibility greatly increases the possibilities with regards to call routing mangement and makes it feasible to consider optimizing the routing with respect to some performance measure. Such a situation creates the need for new routing models which allow for the optimization of network performance through improved switching rules.

This paper focuses on the comparison of two optimized routing policies for circuit switched networks: a centralized and a decentralized one. We introduce here a decentralized optimization model and compare it with a centralized optimization model already presented by the authors (see Sansó, Soumis and Gendreau (1990)). The comparative approach has been inspired by the traditional opposition in transportation networks between system-optimized and user-optimized flows. To recall briefly, in a transportation context, each user is faced with the problem of finding the minimum cost traveling route to reach his destination in a congested network. The users of the network thus compete for a scarce resource, in this case, the capacity of the network. If users behave in a cooperative fashion, a situation may be reached in which the total traveling cost of users is minimized (a system-optimized solution), while if real competition takes place, the aggregate behavior of all users usually leads to some kind of equilibrium in which no one may improve his traveling cost (a user-optimized solution).

In telecommunication networks, we have prefered to use the terms *centralized* and *decentralized* to indicate a cooperative or competitive routing behavior. In fact, whereas in transportation networks users may have the possibility of deciding which route to take, in a telecommunication system they do not have this possibility; the routing of a call is determined by the system's management. The notions of centralized and decentralized optimization are not new in telecommunications. In the field of packet switching, Fratta, Gerla and Kleinrock (1973) posed the system optimal static routing problem as a convex

programming problem and introduced the flow deviation algorithm. The same trend was followed by Gallager (1977) on quasi-static routing problems. Gavish and Hantler (1983) tackle the problem of route selection as a non-linear combinatorial optimization problem. Mason(1985) described analytic models for user-optimized adaptive routing and calculated off-line the system optimum routing patterns. In circuit switched networks Benes (1966) presented the first rigorous approach to optimal routing. Mason and Girard (1982) presented an unifying framework for a number of routing techniques and evaluated performance models by comparing them with exact Markov chain analysis, they discussed the issues of system vs user optimization.

Traditionally, the routing of calls has been characterized by three main stages (see Girard (1990) as reference): (i) identification of the available paths to route a call, (ii) selection of a path to be taken by a call, (iii) management of blocked calls. In the routing schemes which are presented in this paper, these three stages are incorporated into a single one, in other words, the routing policies presented give an answer to the three points at the same time.

The routing schemes that we present have been formulated to optimize some measure of network performance in the network. A centralized optimization routing approach implies the existance of a central decision system that, with knowledge of the state of the system, will allocate the ressource, in this case the system capacity, so as to maximize the global performance of the network. A decentralized optimization approach considers the network as partitioned in local decision systems, each system will have knowledge of the network state but will try to allocate the capacity so as to optimize its local performance.

Details about the measure of performance considered can be found in Sansó (1988) and Sansó, Soumis and Gendreau (1990) None of the two models presented has been actually implemented in a real network. They constitute, however, a useful theoretical approximation of two types of routing policies that could be implementable in the future.

The paper is organized as follows. In section 2, we recall the most important features of the centralized optimization model. In section 3 we introduce the decentralized optimization model. The theoretical relationship between the two models is outlined in section 4 while the models are compared on the basis of a computational study in section 5. Comparative performance results are presented in section 6.

2 The Centralized Optimization Model

In this section, we introduce the stochastic centralized optimization model. This model was first described in Sansó (1988) and a particular application of it was presented in Sansó, Soumis and Gendreau (1990); however, since many modelling details are common with the decentralized model and for the sake of completeness in the theoretical and practical comparison between the two models, its most relevant features will be briefly recalled. The reader interested in further details can find them in the previously mentioned references.

In the routing schemes that are found in the literature (see, for instance Girard (1990), Ott and Krishnan (1988), Mason and Girard (1982), Deschamps (1979), Benes (1966)), the optimization of the network performance is usually an explicit or implicit objective. The performance measure commonly used is the global probability of blocking in the network. As it will be seen later on, this is in most cases determined by Erlang's B formula. This function is non-convex with respect to the traffic variable, and therefore the use of this formula in a centralized optimization problem leads to a non-convex problem, very difficult to solve. For this reason a different approach was chosen in the proposed model.

We first describe the general philosophy of the model. There is a demand, in terms of calls that have to be connected, from every origin to every destination. This demand of calls from different origin-destination pairs of the network will compete for the capacity available. If the demand is such that the calls cannot get through the network, the calls will simply be lost. In the centralized or cooperative scheme of this present model, the competition is managed by a central entity. Calls are routed in a way such as to minimize the number of calls that are lost, globally, in the network.

This problem is a multicommodity flow problem with capacity constraints. Such problems are extremely difficult to solve (for a wide review on multicommodity problems, see Assad (1978)). One common technique used to solve those problems is to eliminate the capacity constraints and add, instead, a penalty term to the objective function. In this way, the problem becomes a nonlinear easy to solve problem.

Sansó (1988) proposed a penalty function that not only allows the capacity con-

straints to be dropped, thus converting a difficult problem into an easy one, but also measures a factor that one would naturally want to minimize in a communication network: the lost call traffic. This function, Erlang's lost call traffic function, has the additional advantage of taking into account the stochastic character of the demand. Therefore, we not only have a problem much easier to solve than the original one, but also one that is closer to reality.

In order to have all the elements to define this first optimization model, we introduce some necessary notation:

N: node set for the entire graph;
M: arc set for the entire graph;
O: origin node set;
C_u: capacity of arc u;
X_u^p: traffic offered to arc u that comes from origin p.

Let $B(s,a)$ be the well known Erlang's blocking probability function, which gives the probability of blocking a call on a link with a capacity of s trunks and a demand, or offered traffic, of a Erlangs. The expression for this blocking probability formula is given by:

$$B(s,a) = \frac{a^s/s!}{\sum_{i=1}^{s} a^i/i!}. \qquad (1)$$

This formula can also be seen as representing the proportion of calls that will be blocked, as a function of the capacity and the link offered traffic. This function has been proven convex with respect variable s (see Messerli(1972),Jagers and Van Doorn(1986)) and the non-convexity of the function with respect variable a has been studied by Harel (1990). Let now $P_u(X)$ be the penalty function chosen, that is Erlang's expected lost call traffic function. This function can be expressed as a function of the probability of blocking as follows:

$$P_u(X) = \sum_{p \in O} X_u^p \; B(C_u, \sum_{p \in O} X_u^p \;). \qquad (2)$$

Note that even though the expected lost call traffic function depends both on the offered traffic and the arc capacity, it is denoted only as a function of the offered traffic, X. This simplification in the notation has been done since only the offered traffic variable X will intervene in the model.

Let Ω represent the set of flows such that flow conservation and nonnegativity constraints are respected. Then the centralized optimization model (COM) can be formulated as follows,

COM:

$$\min_{X \in \Omega} Z = \sum_{u \in M} P_u(X). \tag{3}$$

The objective function of model COM is additive and separable by arc. This function implicitly considers the assumption that the probability that a call is blocked in more than one arc is negligible. In other words, the objective function presented is an upper bound on the real value of the performance measure. The bound is closer to the actual measure when there is little congestion in the network. Another interesting remark is the two different behaviors of variable X. In fact, even though this variable represents the offered traffic, according to the model, it has to respect flow conservation constraints as well. This represents an approximation on the value of variable X. The approximation is closer to reality when there is little congestion in the network. This double behavior of the variable X will be eliminated in the decentralized model that will be formulated in terms of the accepted traffic variable.

The objective function of model COM has been proven to be convex over variable X by Palm (1957), a different proof can be found in Sansó's dissertation (1988). The convexity of the objective function guarantees the convergence of any descent algorithm.

3 The Decentralized Model

In the decentralized model presented in this section, the network is not seen as a cooperative entity in which all calls are routed in order to optimize the global performance but rather as made up of several markets that will compete for the capacity available in order to route their calls. This competing behavior will lead towards an equilibrium in the system. We have used the term *market* in this context to emphasize the idea of competition in the decentralized model. A market, can be, in fact, any origin-destination pair of nodes of the system.

At this point, it is important to place the decentralized and centralized routing models with respect to the state of the art in routing technology. Actual routing models are mostly constituted by rules on where to route a call. This rules are applied at the switching station level and, in general, do not explicitly consider any type of optimization policy. The decentralized model to be presented corresponds to a specific switching rule, however, it does explicitly optimize local market performance. Even though this type of model is not implementable at the moment in telecommunication networks, it could be among the first types of optimization models to be put in practice with the advance of switching technology.

We now pass to the description of the routing features of the model. Let R be the set of markets in the system. Then, for every market $r \in R$, there is a choice of n routes to assign the demand of the market, d_r. The possible routes that can be taken for a particular market r are represented in the set Q_r. The demand of the market, will be assigned to *all* routes of the market according to an attraction coefficient A_q such that:

$$\sum_{q \in Q_r} A_q = 1. \tag{4}$$

The portion of the demand that is refused, tries again. In other words, it is re-assigned on the n routes according to the route attraction coefficients. This re-assignement is done *regardless* of what route a particular call tried the previous time. This has been assumed to make the model easier to implement and to analyze.

In order to be able to express the model behavior in mathematical terms, we need

some additional notation:

U_q: set of arcs belonging to route $q \in Q_r$;
X_q: offered traffic to route $q \in Q_r$;
H_q: traffic accepted on route $q \in Q_r$;
X: vector representing the offered traffic in all routes of the network;
H: vector representing the accepted traffic in all routes of the network;
$\mathcal{P}_q(X)$: lost call traffic on route $q \in Q_r$ as a function of the network offered traffic X.

It should be pointed out that, for this model, X represents the route offered traffic vector whereas in the previous model it represented the arc offered traffic vector. The same notation was kept since it does not lead to confusion and it does clearly identify the nature of the variable for the two models.

Under the behavior described for the decentralized model, variable X_q, which represents route q offered traffic, must be proportional to the attraction coefficient given to the route, A_q and there must be a variable u_r for every $q \in Q_r$ such that

$$X_q = u_r A_q, \quad \forall r \in R. \tag{5}$$

One also has

$$X_q = H_q + \mathcal{P}_q(X). \tag{6}$$

The offered traffic on route q is split in accepted traffic and overflow. We now present the relationship between the overflow on route q, $\mathcal{P}_q(X)$, and the lost call trafffic on a particular arc of the route.

Let X_u^r represent the offered traffic of market r that is trying to pass through arc $u \in U_q$ and let X_u represent the total offered traffic on arc u, that is:

$$X_u = \sum_{r \in R} X_u^r. \tag{7}$$

The overflow for route q can then be expressed as:

$$\mathcal{P}_q(X) = \sum_{u \in U_q} (X_u^r / X_u) P_u(X). \tag{8}$$

This equation implicitly considers the same assumption made for model (COM), that is, that the probability of having a call blocked more than once in the same route is negligible. In addition, the equation holds only if one supposes that there is no discrimination with respect to the call nature for the routing or blocking of a call: the blocking probability of a particular call is independent of its nature.

Let us suppose that it is possible to find a function of the accepted traffic that is the inverse of the network offered traffic, that is:

$$s(H) = X, \tag{9}$$

then the decentralized model (*DOM*) can now be formulated in terms of the accepted traffic variables.

(DOM):

$$\frac{H_q + \mathcal{P}_q(s(H))}{A_q} = u_r, \qquad \forall r \in R \tag{10}$$

$$\sum_{q \in Q_r} H_q = d^r. \tag{11}$$

In the next section we will explore the possibility of solving this model with the same type of algorithm that will be used to solve the centralized model.

4 Relationship between the Centralized and the Decentralized model

One way of solving the system of nonlinear equations describing the decentralized model (DOM) is by assuming that those equations may correspond to the optimality conditions of some centralized optimization problem. This approach, which has been widely used to solve user's optimized models in transportation, would be of particular interest for our purpose of comparing the two type of models. In fact, if model (DOM) had some equivalent centralized optimization model, the same solution algorithm could be used to solve (COM) and (DOM) and, therefore, the result comparison would be facilitated.

In our particular problem, the question we have to ask is if there exists an objective $\mathcal{F}$ such that

$$\frac{\partial \mathcal{F}}{\partial H_q} = \frac{H_q + \mathcal{P}_q(s(H))}{A_q}. \tag{12}$$

According to the Fondamental Theorem of Calculus this objective function $\mathcal{F}$ will exist if the following condition is respected:

$$\frac{\partial^2 \mathcal{F}}{\partial H_q \partial H_p} = \frac{\partial^2 \mathcal{F}}{\partial H_p \partial H_q}, \tag{13}$$

this condition is equivalent to

$$A_p \frac{\partial \mathcal{P}_q(s(H))}{\partial H_p} = A_q \frac{\partial \mathcal{P}_p(s(H))}{\partial H_q}. \tag{14}$$

When routes p and q are such that they belong to *the same market* and they do not have any arcs in common, then the condition for the existence of an objective function $\mathcal{F}$ of a centralized optimization problem equivalent to model (DOM) holds. However, when p and q are routes of different markets that may have arcs in common, the previous condition does not hold. Since we are not restricting our model to the

case in which no common arcs are allowed between routes of different markets, then the condition does not hold for the general case. In other words, model (DOM) *does not* have an equivalent centralized optimization problem whose optimality conditions is the set of nonlinear equations defined by (10) and (11). However, the conditions do hold *within* every market and, therefore, the subproblems (COPr) and (DOPr) that follow are equivalent.

(COPr):

$$\min \mathcal{F} \tag{15}$$

$$s.t.$$

$$\sum_{q \in Q_r} H_q = d^r. \tag{16}$$

(DOPr):

$$\frac{H_q + \mathcal{P}_q(s(H))}{A_q} = u_r, \tag{17}$$

$$\sum_{q \in Q_r} H_q = d^r. \tag{18}$$

The equivalence of the two subproblems constitutes the theoretical basis for the cyclic decomposition algorithm used to solve the decentralized model (DOM).

5 Resolution

In this section, we first describe the general algorithm that was used to solve both models. Secondly we explain the specific implementation for each model, in particular, the decomposition procedure that allowed us to solve the decentralized model (DOM).

5.1 The Algorithm

The algorithm used to solve the optimization problems is an adaptation of the Convex Simplex classical method, first presented by Nguyen(1973) and implemented by Sauvé (1984). It takes advantage of the commodity separation and the network structure of the problem and has a special criteria for the entering variable.

Firstly, the variables of the basis are grouped by commodity with each group becoming a spanning tree of the network. Thus, no matrix storage is required for the simplex iterations and only the tree belonging to the entering variable commodity will be used in every iteration.

Secondly, instead of selecting the entering variable as the one with the most negative reduced cost, the selection is made differently. An ϵ_1 is initially set and the entering variable is selected among those variables, of a particular commodity k, whose reduced cost is inferior to ϵ_1.

At each step, ϵ_1 is reduced by a rate λ, $(\lambda < 1)$. When $\epsilon_1 \leq \epsilon_2$, ϵ_2 being fixed in advance, and no improved solution can be found for any commodity, then the procedure terminates.

The gradual reduction of ϵ_1 prevents the algorithm to spend too much time on one commodity if the other commodities have not been adjusted. On the other hand, the value given to λ allows us to modify the algorithm behavior. In fact, if λ is close to 1, ϵ_1 will remain large and the algorithm will behave as a traditional Convex Simplex. On the other hand, if λ is small, ϵ_1 will soon become very close to the stopping criteria ϵ_2 which will make the algorithm spend long time on the optimization of one particular commodity. In other words, the algorithm, for this case, will behave as a decomposition

algorithm. Therefore, a calibration of λ and ϵ_1 is necessary to get an efficient intermediate algorithm.

5.2 The Computational Procedure

Model (COM) was solved with a straightforward adaptation of the Convex Simplex algorithm described. For this model, the commodities were represented by the origin nodes. The variables λ and ϵ_1 were set so that the procedure behaved as an algorithm intermediate between a traditional Convex Simplex and a decomposition. The convergence of the procedure is guaranteed by the convexity of the objective function of model (COM).

For the second model, a Gauss-Seidel type of approach was used; the network was divided in markets, every market representing a commodity. The λ was set small to make the algorithm behave as a decomposition. The procedure, which can then be seen as a cyclic decomposition scheme, adjusted at every stage the variables for each market r, so that the conditions defined by subproblem (DOPr) were reached. The procedure was repeated until no significant change was detected in any of the markets.

The convergence of the Gauss-Seidel method for a set of nonlinear equations is not guaranteed. Ortega and Rheiboldt (1970) have established some conditions to be respected that would guarantee the convergence of the method, however they are very difficult to verify in practice. In general two types of conditions can be identified: a certain symmetry of the Jacobian matrix of the system of equations defined as $f(X) = 0$, f being a convex function (see Hammond and Magnanti (1987)), or the diagonal dominance of the system matrix, even if f is a non-convex function (see Ortega and Rheiboldt). The first condition guarantees a convergence of the objective function of an equivalent optimization problem. The second condition guarantees a convergence on the variables of the problem, even if an equivalent optimization problem doesn't exist. It was shown, in the previous section, that the set of equations defining model DOM has no equivalent optimization problem. Therefore we tackle the convergence problem on the basis of the second type of condition. The condition could not be theoretically verified for the general model. In fact, the conditions depends on the the problem data and it will always be possible to create a problem so that the Gauss Seidel algorithm will not converge or will have multiple solutions.

The problem of multiple solutions of a system of non-linear equations describing a particular routing model or phenomenon it is not a new problem in circuit switched networks. We mention, for instance, the works by Nakagome and Mori (1973) in which it is shown that the system of equations describing the exact flow values of a mutual overflow network can present multiple solutions. Akinpelu (1983) showed that networks with non-hierarchical alternate routing can have bi-stable modes of operation. The stability of circuit switched networks in systems with mutual overflow and non-hierarchical routing was studied by Mason (1986). The study related various parameters to network instability for three different types of non-hierarchical routing models. One of the characterizations of network instability was the non-uniqueness of equilibrium flow patterns. The study concluded that instability is a pervasive phenomenon in networks with mutual overflow and provided means to stabilize otherwise unstable networks.

Despite the fact that convergence conditions could not be verified for a general network, oscillatory or divergent behavior can easily be detected during the solution procedure. One may thus apply a modified version, which checks for such behavior, of the previously defined algorithm: this method will either terminate with an equilibrium solution or a non-convergent condition. In general, the oscillatory behavior will take place when there is a strong interaction between markets, in other words, when mutual overflow is important. On the other hand, if an equilibrium is reached, the unicity of the equilibrium point is guaranteed by the fact that function $\mathcal{F}$ is strictly increasing.

6 Results

The network used to evaluate the two types of models was a hypothetical network of 40 nodes and 62 arcs. The implementation for the centralized model required the handling of 5960 variables and 40 commodities. The implementation of the decentralized model needed the handling of 4800 variables and 1600 commodities.

The behavior of the centralized and the decentralized models was evaluated for different levels of demand. The figure depicts the lost call traffic for different levels of demand, for the two models.

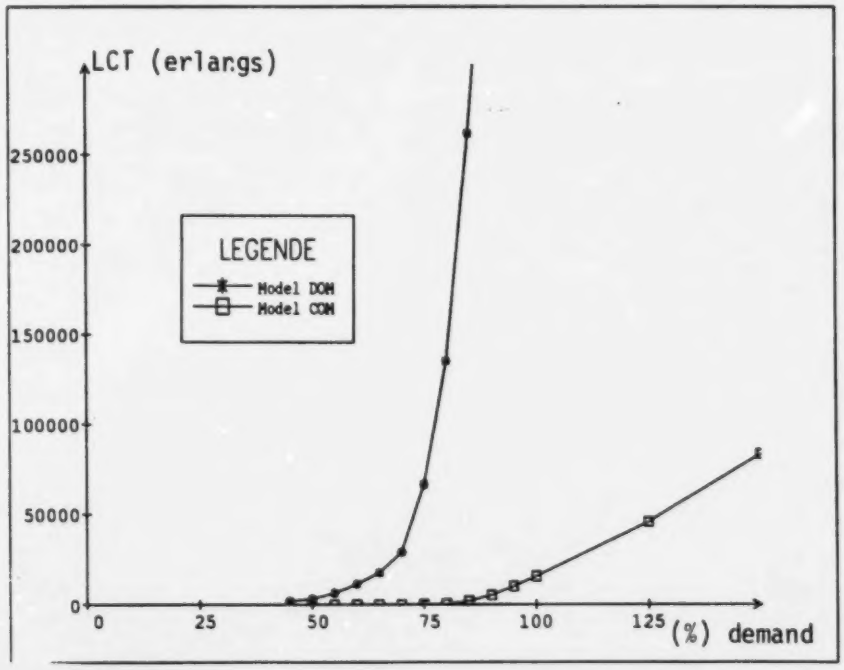

Figure 1: Lost call traffic vs demand for the centralized and decentralized models

The figure shows that for small levels of demand, both models are almost equivalent. On the other hand, the decentralized model reaches an important level of congestion much earlier than the centralized model. It can be seen that the decentralized model

presents a vertical asymptote when the network is too congested. In other words with a decentralized type of routing such as the one considered, the network could not handle an important increase on the level of the demand. On the other hand, the centralized model also presents an asymptotical behavior, but toward a line of slope 1. This is due to the fact that, for greater levels of demand the assumption made regarding the additivity of the objective function of model (COM) is no longer valid. In fact, if it were possible to formulate model (COM) in terms of the accepted traffic rather than the offered traffic, the model would present a vertical asymptote like the one obtained for model (DOM), but this time, the asymptote would occur for higher levels of congestion.

The comparison between model (COM) and model (DOM), is therefore particularly valid for intermediate levels of congestion, that is if the values of variables X are H are similar.

In table 1 we present total execution times for the two models for different levels of demand. The times given are total execution times in seconds in a SUN 3/50.

TABLE 1

Level of demand (%)	time (COM) (sec)	time (DOM) (sec)
45	12.1	625
50	12.2	667
55	12.9	729
60	13.9	853
65	14.9	998
70	17.7	1190
75	20.2	1301
80	29.2	1437
85	37.2	1675
90	46.7	2086
95	46.9	1866
100	38.7	1862
125	41.7	2100
150	44.5	2173

As expected, we observe that the decentralized model takes more than 40 times more time to solve than the centralized one. This is due to the fact that the decentralized model has been formulated in terms of origin- destination pairs as being the commodities whereas the centralized one was formulated in terms of the origin nodes as being the commodities.

7 Summary and Conclusions

In this paper, we have presented and compared two different routing policies, one corresponding to a centralized optimization approach and the other corresponding to a decentralized approach. The decentralized model was being presented for the first time. This approach, although still far from being implemented in real networks, constitutes a type of routing, intermediate between centralized optimization and simple routing rules, that could be implementable in a near future.

The two models were compared on the basis of theoretical considerations and a practical application. The solution approaches correspond to two different implementations of the same Convex Simplex type algorithm. The centralized optimization model was solved directly, whereas a cyclic decomposition procedure was created to solve the decentralized model by the Gauss-Seidel method. The convergence of the Gauss-Seidel decomposition could not be guaranteed, in fact, since it depends greatly on the problem configuration, it will always be possible to create non-convergent examples. However, a procedure was implemented to detect divergent or oscillatory behavior in practice and in our example none of the cases run presented a non-convergent behavior.

A practical application was presented. The application was created from real data and the size of the problem corresponds to that of a real network. Both models were solved quickly on a SUN 3/50. However, as could be expected, the decentralized model was more than 40 times longer to solve than the centralized one. This difference in the solution times can be traced back to the fact that the first model can be formulated as having one commodity per origin whereas the second must be formulated in terms of individual origin-destination pairs.

The two models were evaluated for a changing level of demand. The performance of the network, in terms of how well it could handle congestion for a particular routing policy, was the measure of comparison. As expected, it was found that the centralized optimization model is more efficient in terms of network performance than the decentralized one.

The objective of any routing policy is firstly, to optimize the way calls are managed in the network, minimize the number of blocked calls and therefore, increase network

efficiency. In the long run, routing has an impact on network design since a better routing policy can decrease design and dimensioning costs. The importance of presenting the two types of models lays on the fact that they both give an idea of what will be the network performance under two different policies before their actual implementation. In this paper, we have restricted ourselves to the comparison of the two models taking into account a short term performance objective, that is no impact on design and dimensioning was studied. We mention, however, that the authors have recently presented an algorithm to optimize Network Dimensioning based on the routing policy and reliability considerations. We refer the interested reader to Sansó, Gendreau and Soumis (1990)

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